Short Papers

Characteristics of Some Asymmetrical Coupled Transmission Lines

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Abstract—**In this paper, expressions for the characteristics of some asymmetrical coupled transmission lines are derived. These include expressions for the capacitances, impedances, and phase velocities of asymmetrical coplanar strips, asymmetric coupled striplines, and asymmetric coupled microstriplines.**

Accuracy of these expressions is compared against available numerical results obtained using various numerical methods.

I. Introduction

Asymmetric coupled transmission lines can be used for many known applications including filters, directional couplers, and impedance matching networks. They can offer added flexibility in the design of various circuits through an additional variable and inherent impedance transforming capability. Properties and applications of various asymmetrical coupled structures have been extensively examined by many authors [1]–[7]. During analyzing MIC's containing asymmetrical coupled transmission lines, a major problem is the obtaining of the primary parameters (the self and mutual inductances and capacitances) as functions of the lines' physical dimensions. In the following section, closed-form expressions for the characteristics of asymmetrical coplanar strips, asymmetric coupled striplines, and asymmetric coupled microstriplines are derived. The approach used here is based on dividing the total capacitance of the coupled lines into various basic capacitances. These basic capacitances are then calculated using the available expressions which were obtained earlier by conformal mapping or other seminumerical techniques.

II. Asymmetric Coplanar Strips

The dividing of the coplanar strip's total capacitance into air and dielectric capacitances is shown in Fig. 1. Using these capacitances, the total line's capacitance may be written as

$$C_{cps} = C_{cps1} + C_{cps2} \tag{1}$$

where C_{cps1} and C_{cps2} represent capacitances for the field in the air and dielectric regions, respectively. The capacitance C_{cps1} is taken as half the total capacitance of our coplanar strips but when replacing the dielectric by air, while C_{cps2} is taken as half the total capacitance of our coplanar strips but when replacing the air by dielectric material with relative dielectric constant $\epsilon_{r2} = \epsilon_r$. Expressions for the capacitances C_{cpsi} $(i=1,2)$ is given as follows [8]:

$$C_{cpsi} = \epsilon_0 \epsilon_{ri} \frac{\kappa(k')}{\kappa(k)} \qquad (i=1,2) \tag{2a}$$

Manuscript received August 30, 1982; revised August 4, 1983

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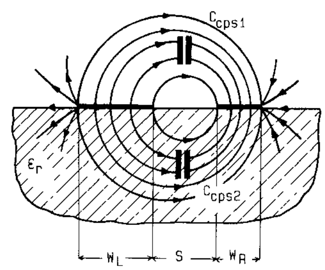

Fig. 1. The dividing of the asymmetrical coplanar strips total capacitance into air and dielectric capacitances.

with

$$k^2 = \frac{1 + W_L/S + W_R/S}{(1+W_L/S)(1+W_R/S)} \tag{2b}$$

and

$$k'^2 = 1 - k^2.$$

Accurate and simple expressions for the ratio $\kappa(k')/\kappa(k)$ are available in [9]. The characteristic impedance and effective dielectric constant are then obtained using the capacitance value given by (1) and (2), and the following relations:

$$Z_0 = \left[c_v \sqrt{CC^a}\right]^{-1} \tag{3a}$$

$$\epsilon_{re} = C/C^a \tag{3b}$$

where c_v is the velocity of light in free space, C is the total line's capacitance, and C^a is the total line's capacitance but when replacing the dielectric by air. It can be easily shown that the effective dielectric constant, in this case, is only a function of the relative dielectric constant and is given by

$$\epsilon_{re(cps)} = \frac{\epsilon_r + 1}{2}. \tag{4}$$

III. Asymmetric Coupled Striplines

The division of the total capacitance of the coupled striplines into parallel-plate, fringe, and gap capacitances is shown in Fig. 2(a). In this case, two modes will be propagating on the lines but with unequal characteristic impedances seen by each of the lines. The characteristic impedances and effective dielectric constants are calculated using the self and mutual capacitances of the lines C_{LL}, C_{RR}, C_{LR}, inductances L_{LL}, L_{RR}, and L_{LR} as well as the following equations from [10]:

$$\epsilon_{re(C,\pi)} = 2c_v^2\left[L_{LL}C_{LL} + L_{RR}C_{RR} - 2L_{LR}C_{LR} \pm \lambda\right]^{-1} \tag{5a}$$

$$Z_{CL} = \frac{c_v}{\sqrt{\epsilon_{reC}}}(L_{LL} - L_{LR}/R_\pi) \tag{5b}$$

$$Z_{\pi L} = \frac{c_v}{\sqrt{\epsilon_{re\pi}}}(L_{LL} - L_{LR}/R_C) \tag{5c}$$

$$Z_{CR} = -R_C R_\pi Z_{CL} \tag{5d}$$

$$Z_{\pi R} = -R_C R_\pi Z_{\pi L} \tag{5e}$$

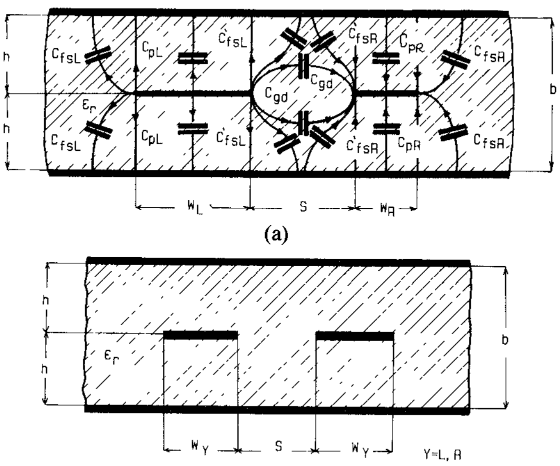

Fig. 2. (a) Division of the total capacitance of the asymmetric coupled striplines into various capacitances. (b) Symmetrical coupled striplines corresponding to our asymmetrical coupled striplines.

where

$$\lambda = \sqrt{4(L_{LR}C_{RR} - L_{LL}C_{LR})(L_{LR}C_{LL} - L_{RR}C_{LR}) + (L_{RR}C_{RR} - L_{LL}C_{LL})^2} \tag{6a}$$

and

$$R_{(C,\pi)} = \frac{(L_{RR}C_{RR} - L_{LL}C_{LL}) \pm \lambda}{2(L_{LR}C_{RR} - L_{LL}C_{LR})}. \tag{6b}$$

The self and mutual capacitances C_{LL}, C_{RR}, and C_{LR} may be written in terms of the parallel-plate, fringe, and gap capacitances, using Fig. 2(a), as follows:

$$C_{LL} = 2\left(C_{fsL} + C_{pL} + C'_{fsL} + C_{gd}\right) \tag{7a}$$

$$C_{RR} = 2\left(C_{fsR} + C_{pR} + C'_{fsR} + C_{gd}\right) \tag{7b}$$

$$C_{LR} = 2C_{gd} \tag{7c}$$

where C_{pY} $(Y = L, R)$ are the left and right parallel-plate capacitances and are given by

$$C_{pY} = \epsilon_0 \epsilon_r \frac{W_Y}{h} \qquad (Y = L, R). \tag{8}$$

The expressions for the fringe capacitances C_{fsL}, C_{fsR}, C'_{fsL}, and C'_{fsR} in terms of known quantities are given as follows:

$$C'_{fsY} = C_{esY} - C_{pY} - C_{fsY} \tag{9a}$$

$$C_{fsY} = 0.5\,(C_{sY} - C_{pY}). \tag{9b}$$

An expression for the capacitance C_{gd} is assumed as follows:

$$C_{gd} = \sqrt{C_{gdL}C_{gdR}} \tag{10a}$$

where

$$C_{gdY} = 0.5\,(C_{osY} - C_{esY}) \qquad (Y = L, R). \tag{10b}$$

The capacitances C_{osY}, C_{esY} $(Y = L, R)$ are the odd- and even-mode capacitances of the corresponding symmetrical coupled striplines with the same values of physical dimensions but with the strip width being W_Y $(Y = L, R)$, as shown in Fig. 2(b). These are given by [11]

$$C_{XsY} = \frac{1}{c_v 60\pi} \frac{\kappa(k_{XY})}{\kappa(k'_{XY})} \tag{11}$$

where $X = o, e$ stands for odd- and even-mode. c_v is the velocity of light in free space and $Y = L, R$ stands for the left and right strips. $\kappa(k_{XY})$ and $\kappa(k_{XY})$ are the elliptic function and its complement with

$$k_{XY} = \begin{cases} \tanh\left(\dfrac{\pi}{4}\dfrac{W_Y}{h}\right) / \tanh\left(\dfrac{\pi}{4}\dfrac{S + W_Y}{h}\right), & \text{odd-mode} \\ \tanh\left(\dfrac{\pi}{4}\dfrac{W_Y}{h}\right) \tanh\left(\dfrac{\pi}{4}\dfrac{S + W_Y}{h}\right), & \text{even-mode} \end{cases} \tag{12}$$

where $k'^2 = 1 - k^2$, $(X = o, e$ and $Y = L, R)$.

Simple and accurate expressions for the ratio $\kappa(k)/\kappa(k')$ are available in [9].

The capacitances C_{sY} $(Y = L, R)$ are the capacitances of corresponding single striplines of the same widths W_L and W_R, respectively. They are calculated using (11) but with the $X = s$ standing for single stripline. The module k_{sY} $(Y = L, R)$, in this case, is given by

$$k_{sY} = \tanh\left(\frac{\pi}{4}\frac{W_Y}{h}\right) \qquad (Y = L, R). \tag{13}$$

The self and mutual inductances L_{LL}, L_{RR}, and L_{LR} can be calculated from the self and mutual capacitances C^a_{LL}, C^a_{RR}, and C^a_{LR} when replacing the dielectric material by air [12]

$$L_{LL} = \frac{10C^a_{RR}}{9\Delta C^a}, \quad L_{RR} = \frac{10C^a_{LL}}{9\Delta C^a}, \quad L_{LR} = \frac{10C^a_{LR}}{9\Delta C^a} \tag{14}$$

where $\Delta C^a = C^a_{LL}C^a_{RR} - (C^a_{LR})^2$.

It should be pointed out that in the above equation (14) the capacitance C^a_{LL}, C^a_{RR}, and C^a_{LR} should be in pF/cm in order to obtain the inductances L_{LL}, L_{RR}, and L_{LR} in nH/cm.

Very good agreement is observed between the calculated results using this section's expressions and those reported by Linnèr [13].

IV. Asymmetrical Coupled Microstriplines

The dividing of the total capacitance of the asymmetrical coupled microstriplines into parallel-plate, fringe, and gap capacitances is shown in Fig. 3. Using these capacitances, the self and mutual capacitances may be written as follows:

$$C_{LL} = C_{fL} + C_{pL} + C'_{fsL} + C_{gd} + C_{ga} \tag{15a}$$

$$C_{RR} = C_{fR} + C_{pR} + C'_{fsR} + C_{gd} + C_{ga} \tag{15b}$$

$$C_{LR} = C_{gd} + C_{ga}. \tag{15c}$$

All the above capacitances have been given before by (8)–(13) except that the capacitances C_{fY} $(Y = L, R)$ are the fringe capacitances of corresponding single microstriplines of widths W_L and W_R, respectively. Expressions for C_{fY} $(Y = L, R)$ are given as follows:

$$C_{fY} = 0.5\,(C_{mY} - C_{pY}) \qquad (Y = L, R) \tag{16}$$

where C_{pY} $(Y = L, R)$ have been given before by (8). The capacitance C_{mY} is the total capacitance of the single microstrip line and is given as follows:

$$C_{mY} = \frac{\epsilon_{re(m)}(W_Y, h, \epsilon_r)}{c_v Z_m(W_Y, h, \epsilon_r = 1)}. \tag{17}$$

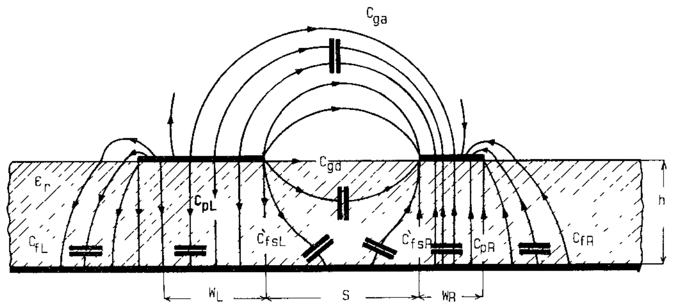

Fig. 3. Division of the total capacitance of the asymmetric coupled microstriplines into various capacitances.

TABLE I
COMPARISON OF THE MODE IMPEDANCES FOR $\epsilon_r = 9.6$ AND $W_L/W_R = 0.6/1.2$ mm

S	Reference [7]				This method			
	Z_{CL}	Z_{CR}	$Z_{\pi L}$	$Z_{\pi R}$	Z_{CL}	Z_{CR}	$Z_{\pi L}$	$Z_{\pi R}$
0.1	75.50	43.90	35.00	20.70	74.50	42.15	34.45	19.49
0.2	71.43	42.86	39.64	24.30	70.81	41.50	38.85	22.77
0.3	68.57	42.14	42.82	26.43	68.04	40.90	41.58	25.05
0.4	66.43	41.43	44.29	27.86	65.90	40.40	43.51	26.69
0.5	64.29	40.71	46.43	29.29	64.26	40.01	44.96	27.99
0.6	63.21	40.00	47.50	30.35	62.99	39.67	46.10	29.04

Reference [7] results consider the effect of frequency dispersion at 10 GHz. S are in millimeters and all impedances are in ohms.

The following expressions for $Z_m(W_Y, h, \epsilon_r = 1)$ and $\epsilon_{re(m)}(W_Y, h, \epsilon_r)$ are available in [14]:

$$Z_m(W_Y, h, \epsilon_r = 1) = \begin{cases} 60 \ln\left[\dfrac{8h}{W_Y} + \dfrac{W_Y}{4h}\right], & W_Y/h \leqslant 1 \\ 120\pi \Big/ \left[\dfrac{W_Y}{h} + 1.393 + 0.677 \ln\left(\dfrac{W_Y}{h} + 1.444\right)\right], & W_Y/h \geqslant 1 \end{cases} \quad (18)$$

$$\epsilon_{re(m)}(W_Y, h, \epsilon_r) = 0.5[(\epsilon_r + 1) + (\epsilon_r - 1) F(W_Y/h)] \quad (19a)$$

where

$$F(W_Y/h) = \begin{cases} (1 + 12h/W_Y)^{-1/2} + 0.04(1 - W_Y/h)^2, & W_Y/h \leqslant 1 \\ (1 + 12h/W_Y)^{-1/2}, & W_Y/h \geqslant 1. \end{cases} \quad (19b)$$

The capacitance C_{ga} represents the gap capacitance in air, and may be calculated by using the results from the capacitance of the corresponding asymmetrical coplanar strips of widths W_L, W_R, and spacing S and with air as a dielectric material, then subtracting the contribution due to the fringe capacitance in air. This contribution may be considered rigorously using the dielectric filling factor defined by Wheeler [15]. However, a value which simplifies the final expression for the air-gap capacitance may be written as follows:

$$\Delta C_f = \left(C_{fL}^a - C_{fsL}^a\right)\left(C_{fR}^a - C_{fsR}^a\right) / \left(C_{fL}^a + C_{fR}^a - C_{fsL}^a - C_{fsR}^a\right). \quad (20)$$

The expression for the gap capacitance C_{ga} in the air may then be written as follows:

$$C_{ga} = 0.5\, C_{cps} - \Delta C_f \quad (21)$$

where C_{cps} is calculated using (1) and (2) and C_{fL}^a, C_{fR}^a, C_{fsL}^a, and C_{fsR}^a are calculated using (16) and (9b), respectively, but with air as dielectric material, i.e.; $\epsilon_r = 1$. A typical set of calculations for $\epsilon_r = 9.6$ using (15)–(21) are compared with values of [7] in Table I.

It has been observed that Jansen's results [7] are a bit higher. This is, of course, due to the fact that Jansen's [7] results consider the effect of frequency dispersion at 10 GHz and for $\epsilon_r = 9.7$.

V. CONCLUSION

Closed-form expressions for the computer-aided design of some asymmetrical coupled microwave integrated transmission lines are derived. These include the asymmetrical coplanar strips, the asymmetric coupled striplines and the asymmetric coupled microstriplines. The derivation is based on dividing the total capacitance of the lines into various basic capacitances that can be calculated using expressions which were obtained earlier by conformal mapping or other seminumerical techniques.

During these derivations, we have corrected the expression for the total capacitance of the asymmetrical coplanar strips and an expression for the gap capacitance of the asymmetric coupled striplines is assumed. The accuracy of the derived expressions has been checked against available numerical results. It may be pointed out that for the asymmetrical coplanar strips, the substrate thickness is assumed to be infinite and for the asymmetric coupled striplines, the two ground-planes are of infinite widths.

It may be also pointed out that for all configurations the thickness of the strips is assumed zero.

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